Nat. Bur. Standards Appl. Math. Ser. No. 37, *Tables of Functions and of Zeros of Functions*, 1954, p. 57–111. See RMT **392**, *MTAC*, v. 2, 1946–47, p. 272; and RMT **104**, *MTAC*, v. 10, 1956, p. 249–250.

2. L. Fox, *The Use and Construction of Mathematical Tables*, National Physical Laboratory Mathematical Tables, v. 1, London, 1956. See RMT **8**, *MTAC*, v. 13, 1959, p. 61–64.

3. L. Fox, *Tables of Everett Interpolation Coefficients*, National Physical Laboratory Mathematical Tables, v. 2, London, 1958.

50[L].—F. W. J. Olver, Editor, *Bessel Functions*, Part III, *Zeros and Associated Values*, Royal Society Mathematical Tables No. 7, Cambridge University Press, New York, 1960, lx + 79 p., 29 cm. Price $9.50.

The present volume is a step towards the completion of a program for the tabulation of Bessel functions initiated by the British Association Mathematical Tables Committee, and continued since 1948 by the Royal Society Mathematical Tables Committee. Part I of this series, *Bessel Functions, Functions of Order Zero and Unity* appeared in 1937, and Part II, *Bessel Functions, Functions of Positive Integer Order* appeared in 1952 (see *MTAC* v. 7, 1953, p. 97–98). Recall that Part I contains a section on the zeros of $J_n(z)$, $Y_n(z)$, $n = 0, 1$, but Part II is without a section devoted to zeros.

Part III, the present work, deals with the evaluation of zeros of the Bessel functions $J_\nu(z)$ and $Y_\nu(z)$ for general ν and z. Tables are also provided as described later in this review. A history of the project is given in the "Introduction and Acknowledgements," by C. W. Jones and F. W. J. Olver. A chapter on "Definitions, Formulae and Methods" by the above authors is a valuable compendium of techniques for the enumeration of zeros and associated functions. In particular, it is an excellent guide if zeros are required of other transcendental functions which satisfy second-order linear differential equations. Several methods of computation are outlined. For instance, the method of McMahon is useful for ν fixed and z large, while the inverse interpolation approach of Miller and Jones presupposes a tabulation of the functions themselves. Between the regions covered by these techniques is a gap which increases with increasing ν. The gap is bridged by application of Olver's important contributions on uniform asymptotic expansions of Bessel functions.

The section "Description of the Tables, Their Use and Preparation" is by the editor. A short description of the tables follows. Table I gives zeros $j_{n,s}$ of $J_n(x)$, $y_{n,s}$ of $Y_n(x)$, and the values of $J_n'(j_{n,s})$, $Y_n'(y_{n,s})$. Table II gives zeros $j'_{n,s}$ of $J_n'(x)$, $y'_{n,s}$ of $Y_n'(x)$, and the values of $J_n(j'_{n,s})$, $Y_n(y'_{n,s})$. Table III gives zeros $a'_{m,s}$, $b'_{m,s}$ of the derivatives $j_m'(x)$, $y_m'(x)$ of the spherical Bessel functions $j_m(x) = (\pi/2x)^{1/2}J_{m+1/2}(x)$, $y_m(x) = (\pi/2x)^{1/2}Y_{m+1/2}(x)$, and the values of $j_m(a'_{m,s})$, $y_m(b'_{m,s})$. The ranges covered are

$$n = 0(\tfrac{1}{2})20\tfrac{1}{2}, \quad s = 1(1)50, \quad \text{Tables I and II;}$$

$$m = 0(1)20, \quad s = 1(1)50, \quad \text{Table III.}$$

All entries are to eight decimals, and in no case should the end-figure error exceed 0.55 of a unit in the eighth decimal.

The coefficients in the uniform asymptotic expansions (previously mentioned) which are used to evaluate items in Tables I–III for n (or m) large are given in Table IV. The expansions for the Bessel functions of Tables I–III also depend on zeros and associated values of certain Airy functions and their derivatives. These data are recorded in Table V. Further description of the contents of Tables IV–V requires much more space and so is omitted here. Suffice it to say that with the aid of these tables, the entries in Tables I–III can with a few exceptions be evaluated to at least eight significant figures for $20 \leqq n < \infty$, $1 \leqq s < \infty$.

There is a good set of references. The printing and typography are excellent, and the present volume upholds the eminent tradition of British table-makers.

Y. L. L.

51[L, V].—J. W. MILES, "The hydrodynamic stability of a thin film of liquid in uniform shearing motion," *J. Fluid Mech.* 8, Pt. 4, 1960, p. 593–610. (Tables were computed by David Giedt.)

Let

$$\mathfrak{F}(z) = [1 - F(z)]^{-1} = w\,[A_i'(-w)]^{-1}\left[\frac{1}{3} + \int_0^w A_i(-t)\,dt\right], \qquad w = ze^{i\pi/6}.$$

$$\mathfrak{F}'(z) = z^{-1}\mathfrak{F}(z) + we^{i\pi/6}[A_i'(-w)]^{-1}A_i(-w)[1 - \mathfrak{F}(z)].$$

$$\mathfrak{F}^{(k)}(z) = \mathfrak{F}_r^{(k)}(z) + i\mathfrak{F}_i^{(k)}(z), \qquad k = 0, 1; \qquad F(z) = Fr(z) + iF_i(z).$$

The paper contains tables of $\mathfrak{F}(z)$, $\mathfrak{F}'(z)$, $F(z)$ and $z^3F_i(z)$ for $z = -6(.1)10$, 4S. The tables were obtained on an automatic computer by numerical integration of an appropriate differential equation. It can be seen from the above that the tables depend on values of the Airy integral $A_i(z)$, its derivative and integral along the rays $\pi/6$ and $-5\pi/6$ in the complex plane. Tables of $A_i(z)$ and its derivative are now available for complex z in rectangular form, but not in polar form. Also, tables of $\int_0^z A_i(\pm t)\,dt$ are available for z real. Thus, the given tables depend on values of some basic functions which, if available, would cut new ground. Unfortunately, the basic items were swallowed up in the automatic computation of $F(z)$. We have here a poor example of table making,—a practice which should not be emulated.

Y. L. L.

52[P].—HELMUT HOTES (Compiler), *Wasserdampftafel der Allgemeinen Elektricitäts-Gesellschaft*, R. Oldenbourg, Munich, 1960, 48 p., 30 cm. DM 16 (Paperback).

There are two tables in this collection. Table I is a four-place table giving the temperature, the specific volume, the specific enthalpy, and the specific entropy as functions of the absolute pressure p. The last three dependent variables are given both for the fluid state and the gaseous state. The variable p ranges from 0.010 to 225,650 atmospheres, and the interval varies from 0.001 to 2000. Table II gives the specific volume, specific enthalpy and specific entropy as functions of temperature for constant pressure. Here p has the values 1, 5, 10 (10) to 400 atmospheres, and t varies from 0 (10) to 330 degrees centigrade.

The tables were calculated by expressing each of the dependent variables as polynomials in the pressure with coefficients as functions of the temperature or in some cases functions of the temperature and pressure. The error bounds given by